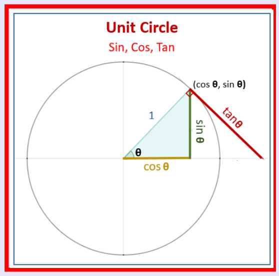
Math 241

Winter 2024

Lecture 5



Feb 19-8:47 AM

Class QZ 3

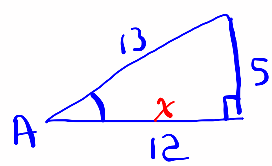
In right triangle ABC, $\sin A = \frac{5}{13}$,

opposite

hypotenuse

Complete the chart below

$\sin A = 5/13$ ✓	$\csc A = 13/5$ ✓
$\cos A = 12/13$ ✓	$\sec A = 13/12$ ✓
$\tan A = 5/12$ ✓	$\cot A = 12/5$ ✓



$x^2 + 5^2 = 13^2$

$x^2 + 25 = 169$

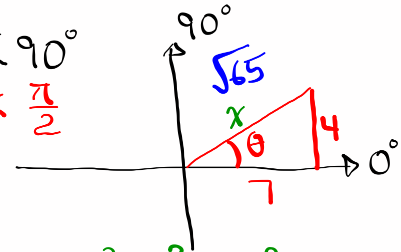
$x^2 = 144$

$\boxed{x = 12}$

Jan 8-12:06 PM

Given $\tan\theta = \frac{4}{7}$ and $0° < \theta < 90°$

$0 < \theta < \frac{\pi}{2}$



$\sin\theta = \frac{4}{\sqrt{65}} = \frac{4\sqrt{65}}{65}$ $\csc\theta = \frac{\sqrt{65}}{4}$

$\cos\theta = \frac{7}{\sqrt{65}} = \frac{7\sqrt{65}}{65}$ $\sec\theta = \frac{\sqrt{65}}{7}$

$\tan\theta = \frac{4}{7}$ $\cot\theta = \frac{7}{4}$

$7^2 + 4^2 = x^2$

$49 + 16 = x^2$

$x^2 = 65$

$x = \sqrt{65}$

$\sin(-\theta) = -\sin\theta = -\frac{4\sqrt{65}}{65}$ $\csc(-\theta) = -\csc\theta = \frac{-\sqrt{65}}{4}$

$\cos(-\theta) = \cos\theta = \frac{7\sqrt{65}}{65}$ $\sec(-\theta) = \sec\theta = \frac{\sqrt{65}}{7}$

$\tan(-\theta) = -\tan\theta = \frac{-4}{7}$ $\cot(-\theta) = -\cot\theta = \frac{-7}{4}$

Jan 9-8:05 AM

Simplify

$\tan^3 x \cdot \csc^3 x$

$= \frac{\cancel{\sin^3 x}}{\cos^3 x} \cdot \frac{1}{\cancel{\sin^3 x}} = \frac{1}{\cos^3 x} = \left(\frac{1}{\cos x}\right)^3$

$= \boxed{\sec^3 x}$

Simplify

$\csc^2 x - \cot^2 x - \cos^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$= 1 + \cancel{\cot^2 x} - \cancel{\cot^2 x} - \cos^2 x$

$= 1 - \cos^2 x = \boxed{\sin^2 x}$

Jan 9-8:15 AM

Simplify

$$\frac{\sec^2\theta - 1}{\cot^2\theta} - \frac{1+\tan^2\theta}{\csc^2\theta - 1}$$

Recall

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$$= \frac{\sec^2\theta - 1}{\cot^2\theta} - \frac{\sec^2\theta}{\cot^2\theta} = \frac{\cancel{\sec^2\theta} - 1 - \cancel{\sec^2\theta}}{\cot^2\theta}$$

$$= \frac{-1}{\cot^2\theta} = -\left(\frac{1}{\cot\theta}\right)^2$$

$$= \boxed{-\tan^2\theta}$$

Jan 9-8:20 AM

Simplify

$$\frac{1}{1-\sec x} + \frac{1}{1+\sec x}$$

LCD $= (1-\sec x)(1+\sec x)$

$$= \frac{1}{1-\sec x} \cdot \frac{1+\sec x}{1+\sec x} + \frac{1}{1+\sec x} \cdot \frac{1-\sec x}{1-\sec x}$$

$$= \frac{1 + \cancel{\sec x} + 1 - \cancel{\sec x}}{(1-\sec x)(1+\sec x)} = \frac{2}{1-\sec^2 x}$$

$(A - B)(A + B)$

$A^2 - B^2$

Recall $1 + \tan^2 x = \sec^2 x$

$$= \frac{2}{1-(1+\tan^2 x)}$$

$$= \frac{2}{\cancel{1} - \cancel{1} - \tan^2 x}$$

$$= -2 \cdot \frac{1}{\tan^2 x}$$

$$= \boxed{-2\cot^2 x}$$

Jan 9-8:25 AM

Expand & Simplify

$$(1 + \cos^2\alpha)(1 + \tan^2\alpha)$$

$$= 1 + \tan^2\alpha + \cos^2\alpha + \cos^2\alpha \cdot \tan^2\alpha$$

$$= 1 + \tan^2\alpha + \cos^2\alpha + \cancel{\cos^2\alpha} \cdot \frac{\sin^2\alpha}{\cancel{\cos^2\alpha}}$$

$$= \sec^2\alpha + \underbrace{\cos^2\alpha + \sin^2\alpha}$$

$$= \boxed{\sec^2\alpha + 1}$$

Jan 9-8:32 AM

Factor, then Simplify

$$\sin^2 x + \cot^2 x \cdot \sin^2 x$$

$$= \sin^2 x \left(1 + \cot^2 x \right)$$

$$= \sin^2 x \cdot \csc^2 x = (\sin x \cdot \csc x)^2 = 1^2 = \boxed{1}$$

Factor $\sec^2 x + 6\tan x + 4$

Hint: what do we know about $\sec^2 x$?

$$= 1 + \tan^2 x + 6\tan x + 4$$

$$= \tan^2 x + 6\tan x + 5$$

$$= \boxed{(\tan x + 1)(\tan x + 5)}$$

Hint: How do we factor

$$A^2 + 6A + 5 = (A+1)(A+5)$$

Jan 9-8:36 AM

Evaluate:

$$\left(\sin^2(25^\circ) - 5 + \cos^2(25^\circ)\right)^3$$

$$= (1 - 5)^3 = (-4)^3 = \boxed{-64}$$

$$5\left(\underbrace{\sec^2(-100^\circ) - \tan^2(-100^\circ)}_{1}\right) - 5$$

Recall

$1 + \tan^2 x = \sec^2 x$

$1 = \sec^2 x - \tan^2 x$

$$= 5 \cdot 1 - 5 = 5 - 5 = \boxed{0}$$

Jan 9-8:44 AM

Simplify

$$2 - \frac{\cos^2 x}{1 - \sin x}$$

$$= 2 - \frac{1 - \sin^2 x}{1 - \sin x}$$

Recall

$A^2 - B^2 = (A+B)(A-B)$

$$= 2 - \frac{(1 + \sin x)\cancel{(1 - \sin x)}}{\cancel{1 - \sin x}} = 2 - (1 + \sin x)$$

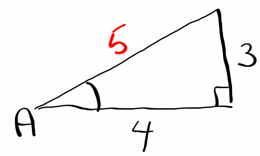
$$= 2 - 1 - \sin x$$

$$= \boxed{1 - \sin x}$$

Jan 9-8:51 AM

Class QZ 4

In right triangle ABC, $\cot A = \frac{4}{3}$, Complete the chart below



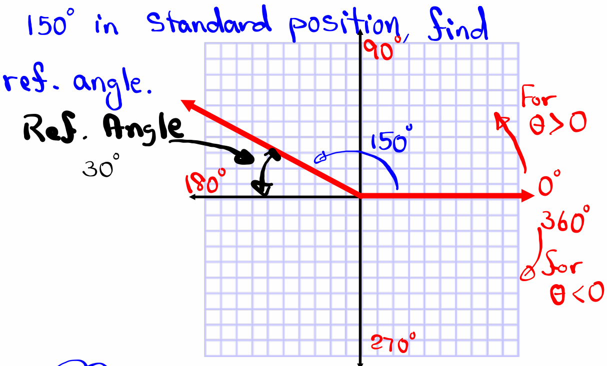
$\sin A = \frac{3}{5}$ $\qquad$ $\csc A = \frac{5}{3}$

$\cos A = \frac{4}{5}$ $\qquad$ $\sec A = \frac{5}{4}$

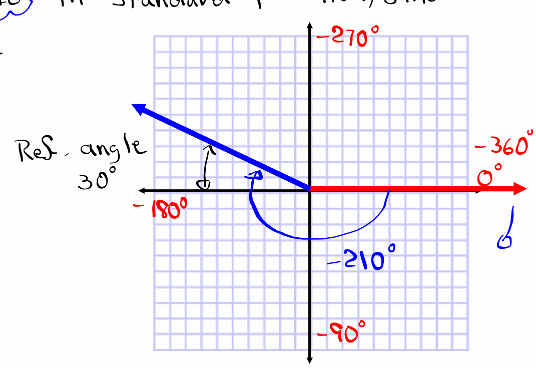
$\tan A = \frac{3}{4}$ $\qquad$ $\boxed{\cot A = \frac{4}{3}}$ Given

Jan 9-8:56 AM

Draw 150° in Standard position, find its ref. angle.



Draw −270° in Standard position, find its ref. angle



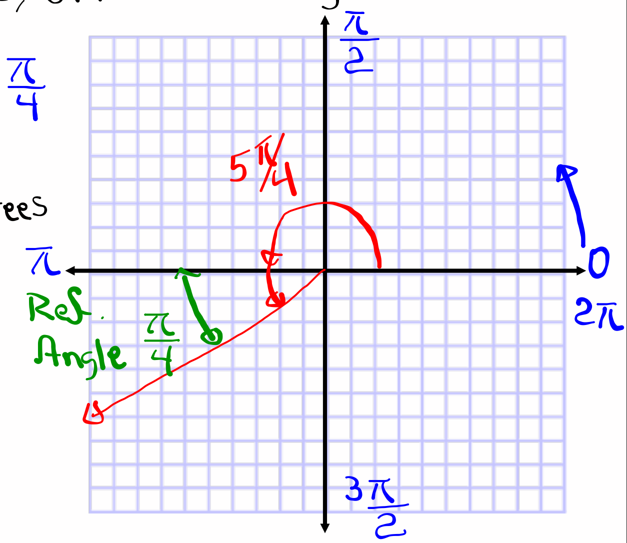
Jan 9-9:24 AM

Draw $\frac{5\pi}{4}$ radians in standard position, Convert it to degrees, find ref. angle in radians.

$$\frac{5\pi}{4} = \frac{4\pi}{4} + \frac{\pi}{4} = \pi + \frac{\pi}{4}$$

$$\frac{5\pi}{4} \text{ Radian} = \frac{5\cancel{\pi}}{\cancel{4}_{1}} \cdot \frac{\cancel{180}^{45}}{\cancel{\pi}} \text{ degrees}$$

$$= \boxed{225^\circ}$$



Jan 9-9:30 AM

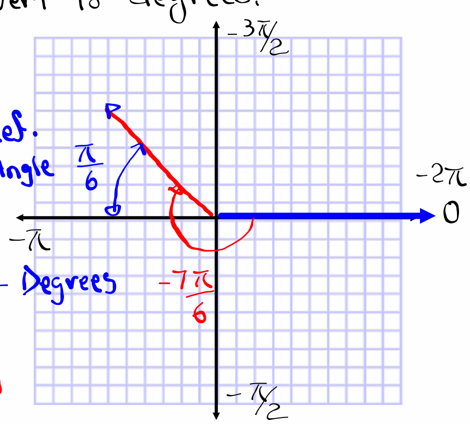
Draw $-\frac{7\pi}{6}$ in standard position, find its ref. angle, Convert to degrees.

$$-\frac{7\pi}{6} = -\frac{6\pi}{6} + \frac{-\pi}{6}$$

$$= -\pi + \frac{-\pi}{6}$$

$$-\frac{7\pi}{6} \text{ Radians} = -\frac{7\cancel{\pi}}{\cancel{6}_{1}} \cdot \frac{\cancel{180}^{30}}{\cancel{\pi}} \text{ Degrees}$$

$$= \boxed{-210^\circ}$$

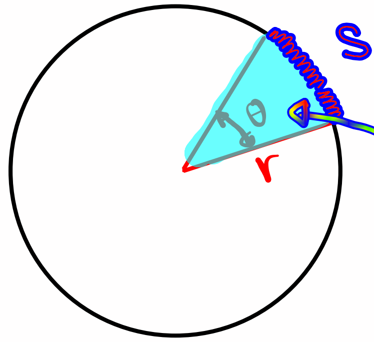


Famous angles!

0°	30°	45°	60°	90°	180°	270°	360°
0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Jan 9-9:36 AM

Sector with Central angle θ



$\boxed{S = r\theta}$

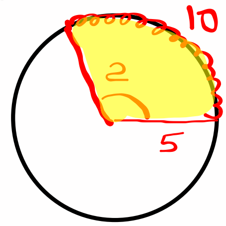
θ must be in radians.

$\boxed{A = \frac{1}{2} r^2 \theta}$

Given $r = 5\text{cm}$ and $\theta = 2$ radians.

$S = r\theta = 5 \cdot 2 = 10\text{cm}.$

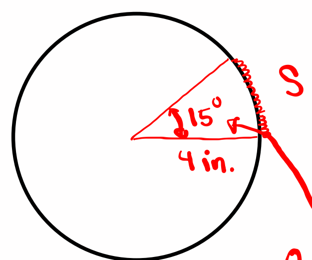
$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \cdot 5^2 \cdot 2 = 25\ \text{cm}^2$



Jan 9-9:44 AM

The central angle of a sector in a circle of radius 4 in. has a measure of 15°.

1) Draw, clearly label.



$\theta = 15°$

$15° = 15 \cdot \frac{\pi}{180}$

$= \frac{\pi}{12}$

2) Find the arc length

$S = r\theta$

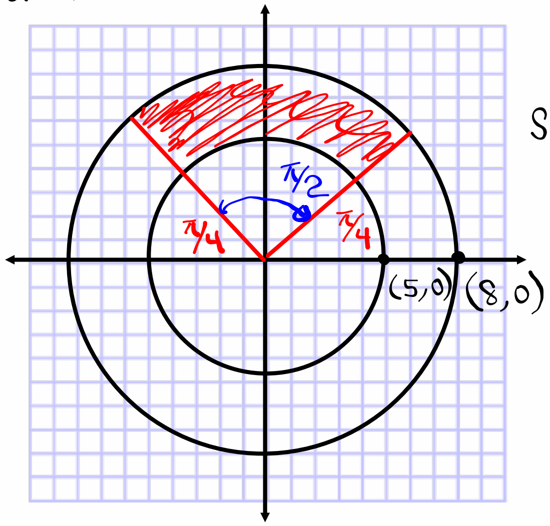
$= 4 \cdot \frac{\pi}{12} = \boxed{\frac{\pi}{3}\ \text{in.}}$

3) Find its area.

$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \cdot 4^2 \cdot \frac{\pi}{12} = \frac{\overset{2}{\cancel{4}} \cdot \cancel{4}\pi}{\cancel{2} \cdot \underset{3}{\cancel{12}}} = \boxed{\frac{2\pi}{3}\ \text{in.}^2}$

Jan 9-9:50 AM

Find the shaded area below



$\theta = \frac{\pi}{2}$

Shaded area = $A_{\text{Larger Sector}} - A_{\text{smaller Sector}}$

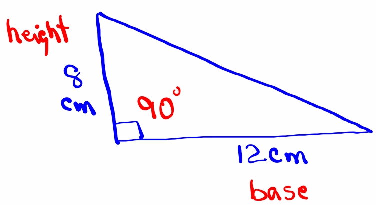
$$= \frac{1}{2}\cdot 8^2 \cdot \frac{\pi}{2} - \frac{1}{2}\cdot 5^2 \cdot \frac{\pi}{2}$$

$$= \frac{64\pi}{4} - \frac{25\pi}{4} = \boxed{\frac{39\pi}{4}}$$

Units^2.

Jan 9-10:00 AM

Find the area of the triangle below

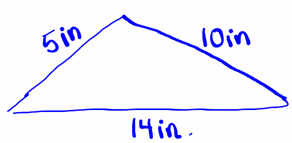


Area = $\frac{1}{2}bh$

$= \frac{1}{2}\cdot 12 \cdot 8$

$= \boxed{48 \text{ cm}^2}$

Find the area of the triangle below



$5^2 + 10^2 \overset{?}{=} 14^2$

$25 + 100 \overset{?}{=} 196$

Not a right Triangle

Use Heron's Formula

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

$s = \frac{5+10+14}{2}$

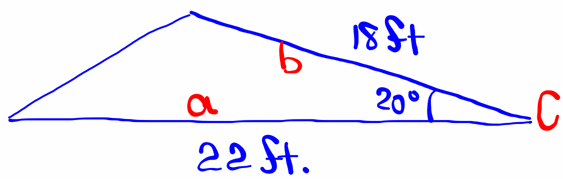
$= 14.5$

$= \sqrt{14.5(14.5-5)(14.5-10)(14.5-14)}$

$= \sqrt{309.9375} \approx \boxed{17.6 \text{ in}^2}$

Jan 9-10:05 AM

Find the area of the triangle below



SAS

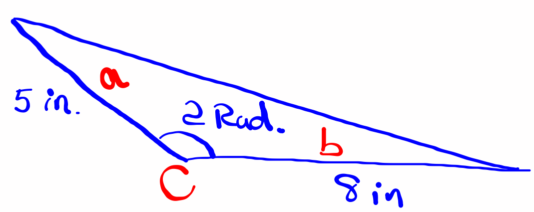
Two sides & angle between them

$$\text{Area} = \frac{1}{2} a b \sin C$$

$$= \frac{1}{2} \cdot 22 \cdot 18 \cdot \sin 20^\circ \approx \boxed{67.7 \text{ ft}^2}$$

Jan 9-10:12 AM

Find the area of the triangle below



Radians

$$\text{Area} = \frac{1}{2} a b \sin C$$

$$= \frac{1}{2} \cdot 5 \cdot 8 \cdot \sin 2 = 20 \sin 2$$

$$\approx \boxed{18.2 \text{ in}^2}$$

Jan 9-10:17 AM

Simplify

$$(3\sin x + 4\cos x)^2 + (3\sin x - 4\cos x)^2$$

$(3A+4B)^2 = (3A+4B)\cdot(3A+4B)$
$= 9A^2 + 12AB + 12AB + 16B^2$

$$= 9\sin^2 x \cancel{+ 24\sin x\cos x} + 16\cos^2 x +$$
$$9\sin^2 x \cancel{- 24\sin x\cos x} + 16\cos^2 x$$

$$= 18\sin^2 x + 32\cos^2 x$$

$$= 18\sin^2 x + 18\cos^2 x + 14\cos^2 x$$

$$= 18(\underbrace{\sin^2 x + \cos^2 x}_{1}) + 14\cos^2 x = \boxed{18 + 14\cos^2 x}$$

Jan 9-10:21 AM

Simplify

$$(3\sin x + 4\cos x)^2 + (4\sin x - 3\cos x)^2$$

$$= 9\sin^2 x \cancel{+ 24\sin x\cos x} + 16\cos^2 x +$$
$$16\sin^2 x \cancel{- 24\sin x\cos x} + 9\cos^2 x$$

$$= 25\sin^2 x + 25\cos^2 x = 25(\sin^2 x + \cos^2 x)$$

$$= 25 \cdot 1$$

$$= \boxed{25}$$

Jan 9-10:21 AM

Verify

$$\frac{1}{\csc x - \cot x} = \frac{1+\cos x}{\sin x}$$

Hint:
Work on LHS
Convert to
$\sin x$ & $\cos x$

LCD = $\sin x$

$$\frac{1}{\csc x - \cot x} = \frac{1}{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}$$

$$= \frac{\sin x \cdot 1}{\cancel{\sin x} \cdot \frac{1}{\cancel{\sin x}} - \cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}}}$$

$$= \frac{\sin x}{1 - \cos x} \cdot \boxed{\frac{\sin x}{\sin x}}$$

$$= \frac{\sin^2 x}{(1-\cos x)\cdot \sin x}$$

use $A^2 - B^2$

$$= \frac{1 - \cos^2 x}{(1-\cos x)\cdot \sin x}$$

$$= \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{(1-\cos x)}\cdot \sin x} = \boxed{\frac{1+\cos x}{\sin x}}$$

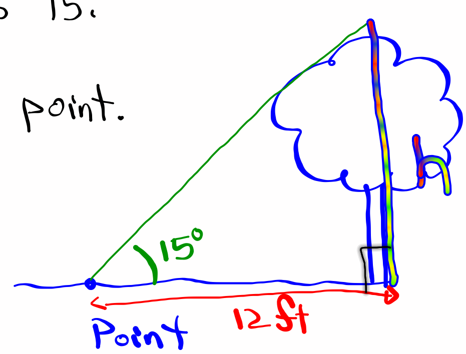
Jan 9-10:34 AM

Angle of elevation from a point on the ground to the top of a tree is 15°.

Tree is 12 ft from that point.

How tall is the tree?

Drawing required.



$$\tan 15° = \frac{h}{12}$$

Cross-Multiply

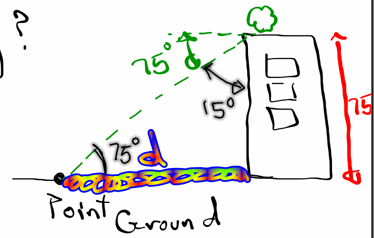
$$h = 12 \tan 15°$$

$$= 3.215 \approx \boxed{3.2 \text{ ft}}$$

Jan 9-11:08 AM

A building is 75 ft tall. Angle of depression to a point on the ground is 75°. How far is the point from the building?

Complete drawing required.



$\tan 75° = \frac{75}{d}$

$d \cdot \tan 75° = 75$

$d = \frac{75}{\tan 75°} = 20.096$

$\approx \boxed{20 \text{ ft}}$

$\tan 15° = \frac{d}{75}$

$d = 75 \tan 15°$

≈ Same answer.

Jan 9-11:14 AM

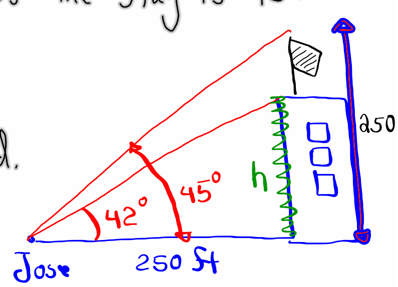
Jose is 250 ft from a building.

There is a flag on top of the building.

Jose's angle of elevation to the top of building is 42°, and to the top of the flag is 45°.

How tall is the flag?

Complete drawing required.



$\tan 42° = \frac{h}{250}$

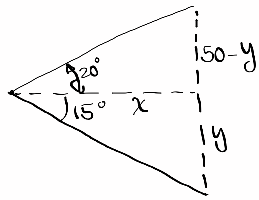
$h = 250 \cdot \tan 42°$

$\approx 225 \text{ ft}$

Flag is $250 - 225 = \boxed{25 \text{ ft}}$ tall.

Jan 9-11:21 AM

Find x using the drawing below



$\tan 20° = \dfrac{50-y}{x}$

$\tan 15° = \dfrac{y}{x}$

$x \tan 20° = 50 - \boxed{y}$

$x \tan 15° = \boxed{y}$

$x \tan 20° = 50 - x \tan 15°$

$x \tan 20° + x \tan 15° = 50$

$x(\tan 20° + \tan 15°) = 50$

$x = \dfrac{50}{\tan 20° + \tan 15°}$

$x \approx 79.124 \approx \boxed{79}$

Calculate $\tan 20° + \tan 15°$ Press x^{-1} now

multiply by 50.

$50 \div (\tan 20° + \tan 15°)$ enter

Jan 9-11:30 AM

Verify

$$\frac{\sin x}{1-\sin x} + \frac{\sin x}{1+\sin x} = 2\tan x \cdot \sec x$$

LCD: $(1-\sin x)(1+\sin x)$

$$\frac{\sin x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x} + \frac{\sin x}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}$$

$$= \frac{\sin x\,[1 \cancel{+\sin x} + 1 \cancel{-\sin x}]}{(1-\sin x)(1+\sin x)} = \frac{2\sin x}{1-\sin^2 x} = \frac{2\sin x}{\cos^2 x}$$

$$= 2 \cdot \boxed{\frac{\sin x}{\cos x}} \cdot \boxed{\frac{1}{\cos x}} = 2\tan x \cdot \sec x$$

Jan 9-11:42 AM

Verify

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$$

Hint:
Factor
$A^3 + B^3$
$= (A+B)(A^2 - AB + B^2)$

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = \frac{\cancel{(\sin x + \cos x)}(\sin^2 x - \sin x \cos x + \cos^2 x)}{\cancel{\sin x + \cos x}}$$

$$= \boxed{1 - \sin x \cos x}$$

Jan 9-11:50 AM

Verify

$$\frac{\sec^2\theta - 6\tan\theta + 7}{\sec^2\theta - 5} = \frac{\tan\theta - 4}{\tan\theta + 2}$$

Hint:
$1 + \tan^2\theta = \sec^2\theta$
Replace $\sec^2\theta$,
Simplify,
Factor,
Simplify
more.

$$\frac{\sec^2\theta - 6\tan\theta + 7}{\sec^2\theta - 5} = \frac{1 + \tan^2\theta - 6\tan\theta + 7}{1 + \tan^2\theta - 5}$$

$$= \frac{\tan^2\theta - 6\tan\theta + 8}{\tan^2\theta - 4} = \frac{(\tan\theta - 4)\cancel{(\tan\theta - 2)}}{(\tan\theta + 2)\cancel{(\tan\theta - 2)}}$$

$$= \frac{\tan\theta - 4}{\tan\theta + 2} \checkmark$$

Jan 9-11:56 AM

Simplify

$$\left(\tan x + \sin^2 x + \cos^2 x\right)\left(\tan x - \sin^2 x - \cos^2 x\right)$$

$$= \left(\tan x + 1\right)\left(\tan x - 1\right)$$

$$1 + \tan^2 x = \sec^2 x$$

$$\tan^2 x = \sec^2 x - 1$$

$$= \tan^2 x - 1$$

$$= \sec^2 x - 1 - 1 = \boxed{\sec^2 x - 2}$$

Jan 9-12:03 PM

Verify

$$\frac{\sec^3 x - \cos^3 x}{\sec x - \cos x} = \sec^2 x + \cos^2 x + 1 \checkmark$$

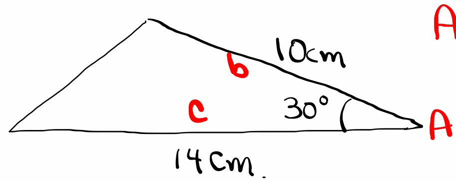
$$LHS = \frac{\cancel{(\sec x - \cos x)}(\sec^2 x + \overbrace{\sec x \cos x}^{1} + \cos^2 x)}{\cancel{\sec x - \cos x}}$$

$$= \sec^2 x + 1 + \cos^2 x \checkmark$$

Jan 9-12:11 PM

Class QZ 5

Find the area of the triangle below



Area $= \frac{1}{2} bc \sin A$

$= \frac{1}{2} \cdot 10 \cdot 14 \cdot \sin 30^\circ$

$= \frac{1}{2} \cdot 10 \cdot 14 \cdot \frac{1}{2} = \boxed{35\text{ cm}^2}$

Jan 9-12:16 PM